# Color Printing System.

Assumption 1. Density of the generated negative [or reversal] image is strictly proportional to log exposure.

Assumption 2. RGB color systems are entirely independent; ie, exposure by one system yields no density change in the other systems.

$$D_i = \gamma_i \log E_i + k_i, \quad i = R, G, \text{ or } B \tag{1}$$

$\gamma$ = contrast slope of the film specs

$k_i$ = film speed constant

A = Table A is a look-up function which converts log E levels to pulse counts. In other words, to effect a particular exposure level using the strobe gun system, one must look up the corresponding strobe count in Table A.

$N_i$ = Exposure level numbers. The preselected range of exposure for each color is logarithmically divided by equal increments, which are numbered sequentially from lowest to highest, starting from zero. $N_i \geq 0$

$$\log E_i = s_i N_i + e_i \ , \quad i = R, G, \text{or } B \tag{2}$$

$s \cong 0.1$ $\quad s_i$ = step size of exposure increments. For $s_i = 0.1$, the step size is about 1/3 stop of exposure

$e_i$ = log of the lowest exposure level to be used.

$N_i$ = exposure level number.

$$A_i(N_i) = P_i \tag{3}$$ where $A$ is the Table A lookup function and $P_i$ is the number of pulses.

Then $D_i = \gamma_i s_i N_i + (\gamma_i e_i + k_i)$, (4)

so that $N_i$ is a linear measure of density.

Assumption 3. The contrast of R, G, and B are equal, and the step sizes of exposure increments for R, G, and B are also equal. Then (4) is rewritten

$$D_i = \gamma s N_i + (\gamma e_i + k_i) \tag{5}$$

We would like to color balance our final prints, so that the steps $N_i$ for red, blue, and green are correlated: If $N_R = N_G = N_B$, the final print should be gray. In (5), only the timing constant $e_i$ is arbitrarily assigned; that is, $N_i$ is a function of $e_i$.

We select the $e_i$ so that equal $N_i$ produce gray

Definition: A color is a set of three values for the $N_i$, notated $(N_R, N_G, N_B)$.

Definition: The saturation $S$ of a color $(N_R, N_G, N_B)$ is a measure of the color's "grayness". The less gray a color, the more saturated it is. To define $S$, we must compare the $N_i$ of the color $(N_R, N_G, N_B)$. Select $N_a$ so that it is less than or equal to any of the other $N_i$:

$$N_a \leq N_b \leq N_c \,.$$

Then

$$S(N_a, N_b, N_c) = \frac{N_b + N_c - 2N_a}{N_b + N_c} \qquad (6)$$

Notice that $N_a = 0$ implies that

$$S = \frac{N_b + N_c}{N_b + N_c} = 1,$$ which is a

maximum value. Also, if $N_a = N_b = N_c$, then

$$S = \frac{0}{N_b + N_c} = 0,$$ minimum saturation.

Gray is the color whose saturation is zero.

Since $S(N_i) = \frac{N_b + N_c - 2N_a}{N_b + N_c} = 1 - 2\frac{N_a}{N_b + N_c}$, saturation is kept constant by fixing the ratio $N_a/(N_b + N_c)$.

Definition: The hue $H$ of a color $(N_a, N_b, N_c)$ is the dominant color comprising that combination. The hue is an ordered triple, made up of a coefficient and two color names:

$$H(N_a, N_b, N_c) = \left(\frac{N_c - N_a}{N_b + N_c - 2N_a}, c, b\right) = \left(h(N_a, N_b, N_c), c, b\right) \qquad (1)$$

where $N_a \leq N_b \leq N_c$.

Notice that an equal change in all $N_i$ does not affect $H(N_i)$:

$$\frac{(N_c + \Delta) - (N_a + \Delta)}{(N_b + \Delta) + (N_c + \Delta) - 2(N_a + \Delta)} = \frac{N_c - N_a}{N_b + N_c - 2N_a}$$

That is, added exposure of gray does not affect line.

If $N_c = N_b = N_a$, $H(N_i)$ is indeterminate (gray), or zero: (0, a,

If $N_c = N_b \neq N_a$, $H(N_i) = (\frac{1}{2}, c, b)$.

If $N_c \neq N_b = N_a$, $H(N_i) = (1, c, [b \text{ or } a])$,

which may be abbreviated $H(N_a, N_a, N_c) = (1, c)$

Except for gray, the numerical value of the first term is $\frac{1}{2} \leq \frac{N_c - N_a}{N_b + N_c - 2N_a} = h(N_i) \leq 1$.

$h(N_i)$ may also be expressed as follows:

$$\frac{N_c - N_a}{N_b + N_c - 2N_a} = \frac{1}{1 + \frac{N_b - N_a}{N_c - N_a}}$$

. That is, H is constant if $(N_b - N_a)$ is proportional to $(N_c - N_a)$.

<u>Definition</u>. The <u>intensity</u> I of a color $(N_a, N_b, N_c)$ is equivalent to the exposure level number which would result from summing the three exposure levels $E_i$

$$I(N_a, N_b, N_c) = \frac{1}{\Delta} \log\left(10^{\Delta N_a} + 10^{\Delta N_b} + 10^{\Delta N_c}\right) \qquad (2)$$

or $I(N_a, N_f, N_c) = N_a + \frac{1}{\Delta}\log\left(10^{\Delta(N_c - N_a)} + 10^{\Delta(N_f - N_a)} + 1\right)$,

which is equivalent. Notice that if $N_a$ is much larger than $N_f$ and $N_c$, then

$$0 < 10^{\Delta(N_c - N_a)} + 10^{\Delta(N_f - N_a)} \ll 1$$

and I will be very close to $N_a$. If the $N_i$ are equal,

$$I(N_i) = N_a + \frac{1}{\Delta}\log 3 = N_a + \frac{0.4771}{\Delta} .$$

Even if the $N_i$ are zero, some contribution is made to intensity; this corresponds to the strictly logarithmic exposure system, in which an exposure of zero corresponds to $N = -\infty$:

If $E = 0$ then $\log E = \Delta N + e = -\infty$

~~Review:~~ ~~S is a function of~~ $\cancel{\frac{N_a}{N_f + N_c}}$

~~H is a function of~~ $\cancel{\frac{N_f - N_a}{N_c - N_a}}$

~~I is a function of~~ $\cancel{\sum_i 10^{\Delta N_i}}$

~~If the hue is constant, then saturation may vary with $N_c$ and $N_f$:~~ $\cancel{\frac{1}{2N-1}\left(\frac{2N_c}{N_f + N_c} - 1\right) = S}$

~~Simulation of Reversal. Hue may be constant on saturation vary as it does in reversal exposure: as $N_i$ is less, saturation increases, but where $N_i$ approaches a maximum, saturation decreases as I passes to a maximum (white).~~

The space of all colors $(N_R, N_G, N_B)$ comprises a set of $n^3$ discrete colors, if $0 \leq N_i < n$. Within this space we find colors which are neighbors.
A color $(N_i, N_j, N_k)$ has six simple neighbors,

$$\begin{array}{ll} (N_i+1, N_j, N_k) & (N_i-1, N_j, N_k) \\ (N_i, N_j+1, N_k) & (N_i, N_j-1, N_k) \\ (N_i, N_j, N_k+1) & (N_i, N_j, N_k-1) \end{array}.$$

It also has the 20 complex neighbors $(N_{i\pm1}, N_{j\pm1}, N_{k\pm1})$, where two or three of the $N_i$ have been changed.
We will now determine the relationships between neighboring colors in terms of hue, saturation, and intensity.
If the hue is a primary color, $N_b = N_a$ and

$$h(N_i) = \frac{1}{1+\frac{N_b - N_a}{N_c - N_a}} = 1, \quad \frac{N_b - N_a}{N_c - N_a} = 0$$

If the hue is a secondary color, $N_c = N_b$ and

$$h(N_i) = \frac{1}{2}, \quad \frac{N_b - N_a}{N_c - N_a} = 1 .$$

When $\frac{N_b - N_a}{N_c - N_a}$ assumes either of these whole-number values, neighbors of $(N_i)$ will exist which have the same hue:

$$h(N_a, N_b, N_c) = \frac{1}{2} \Rightarrow \begin{array}{l} h(N_a \pm 1, N_b, N_c) = \frac{1}{2}, \text{ if } N_a < N_b \\ h(N_a, N_b - 1, N_c - 1) = \frac{1}{2}, \text{ if } N_a < N_b \\ h(N_a, N_b + 1, N_c + 1) = \frac{1}{2} \\ h(N_a \pm 1, N_b - 1, N_c - 1) = \frac{1}{2}, \text{ if } N_a + 1 < N \\ h(N_a \pm 1, N_b + 1, N_c + 1) = \frac{1}{2} . \end{array}$$

In short, Na may change, but $N_b$ and $N_c$ must change togeth

Similarly,

$$h(N_a, N_b, N_c) = 1 \implies \begin{array}{ll} h(N_a+1, N_b+1, N_c) = 1, & \text{if } N_a < N_c \\ h(N_a-1, N_b-1, N_c) = 1, & \text{if } N_a > 0 \\ h(N_a, N_b, N_c \pm 1) = 1, & \text{if } N_c > N_b \\ h(N_a+1, N_b+1, N_c \pm 1) = 1, & \text{if } N_a+1 < N_c \\ h(N_a-1, N_b-1, N_c \pm 1) = 1, & \text{if } N_a > 0 . \end{array}$$

In short, $N_c$ may change, but $N_b$ and $N_a$ must change together. For intermediate hues, $0 < \frac{N_b - N_a}{N_c - N_a} < 1$, increases in $N_a$ will move the hue toward the primary color c; decreases in $N_a$ will move the hue toward the secondary. If there is no neighbor with the same hue, what is the closest hue a neighbor might have? Plotting hue as a function of $(N_b - N_a)$ and $(N_c - N_a)$ we find the following:

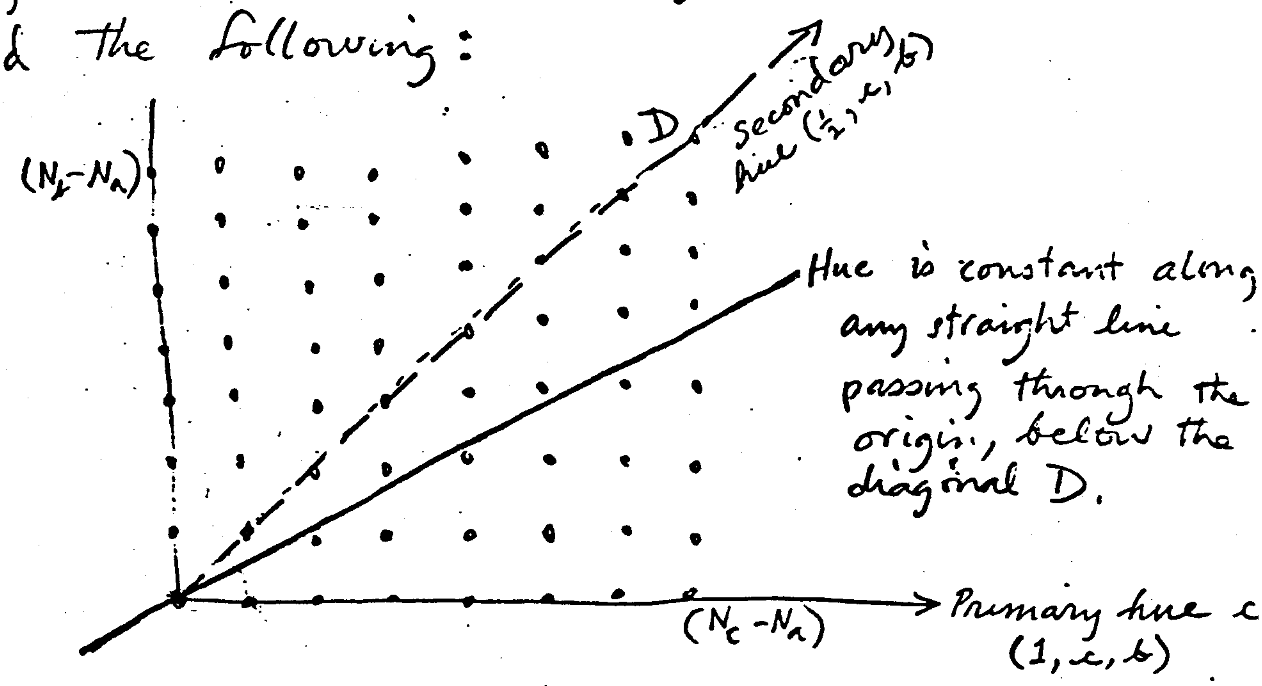


Figure 1

To find neighbors of this sloping line:

Calculate $\frac{(N_b - N_a)}{(N_c - N_a) \pm 1}$ and $\frac{(N_b - N_a) \pm 1}{(N_c - N_a) \pm 1}$. Compare and select the value closer to $\frac{(N_b - N_a)}{(N_c - N_a)}$ of these two.

To follow a best path of constant hue, starting at some gray color corresponding to the origin in Figure 1, select a value for the hue, $h(N_i) = h_0$. Now calculate the ideal value for $\frac{(N_b - N_a)}{(N_c - N_a)}$:

$$h(N_i) = h_0 = \frac{1}{1 + \frac{N_b - N_a}{N_c - N_a}}, \quad \frac{(N_b - N_a)}{(N_c - N_a)} = \frac{1}{h_0} - 1 \qquad (10)$$

Next calculate a list of neighboring colors of hue closest to $h_0$, starting the list with a gray color. To find the next color on the list, form the two values $\frac{(N_b - N_a)}{(N_c - N_a)+1}$ and $\frac{(N_b - N_a)+1}{(N_c - N_a)+1}$. Compare them to $\left(\frac{1}{h_0} - 1\right)$ and select the one that is closest. Now add any color to the list which will result in the chosen new value for $\frac{(N_b - N_a)}{(N_c - N_a)}$. Notice that there will be two choices for each neighbor:

$\frac{(N_b - N_a)}{(N_c - N_a)+1}$ results from $(N_a - 1, N_b - 1, N_c)$ or $(N_a, N_b, N_c + 1)$

$\frac{(N_b - N_a)+1}{(N_c - N_a)+1}$ results from $(N_a - 1, N_b, N_c)$ or $(N_a, N_b + 1, N_c + 1)$

Of the two choices, each of which will have the same hue $h \cong h_o$, both are more saturated than the last value on the list; and one is more saturated than the other:

$$S(N_a-1, N_b-1, N_c) = \frac{(N_b-1)+N_c-2(N_a-1)}{(N_b-1)+N_c} = \frac{(N_b+N_c-2N_a)+1}{(N_b+N_c)-1} > S(N_a, N_b, N_c$$

$$S(N_a, N_b, N_c+1) = \frac{N_b+(N_c+1)-2N_a}{N_b+(N_c+1)} = \frac{(N_b+N_c-2N_a)+1}{(N_b+N_c)+1} \geq S(N_a, N_b, N_c$$

$$S(N_a-1, N_b, N_c) = \frac{N_b+N_c-2(N_a-1)}{N_b+N_c} = \frac{(N_b+N_c-2N_a)+2}{(N_b+N_c)} > S(N_a, N_b+1, N$$

$$S(N_a, N_b+1, N_c+1) = \frac{(N_b+1)+(N_c+1)-2N_a}{(N_b+1)+(N_c+1)} = \frac{(N_b+N_c-2N_a)+2}{(N_b+N_c)+2} \geq S(N_a, N_b, N$$

In short, increases in $(N_c-N_a)$ also increase saturation, for a given hue*.

In making the choices leading to this list of colors, it must be mentioned that $N_a-1$ must not be negative, nor $N_c+1$ greater than $n$, the highest value for $N$.

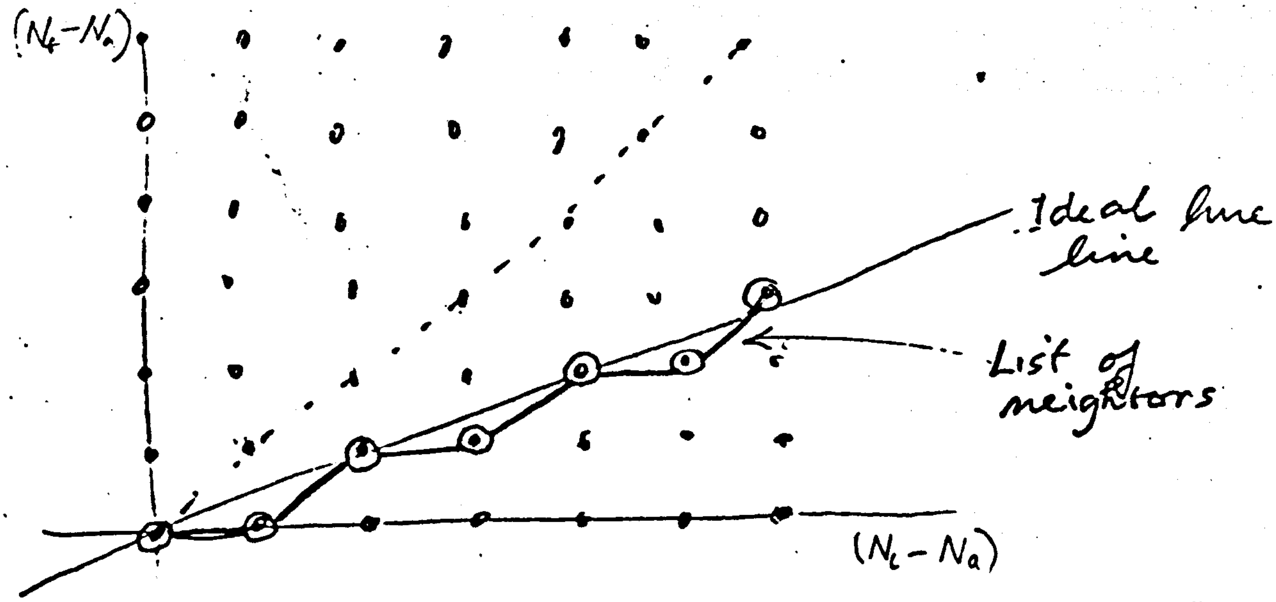


Figure 2

Notice that saturation may be expressed as follows:

$$S(N_i) = \frac{N_b + N_c - 2N_a}{N_b + N_c} = \frac{\frac{N_b - N_a}{N_c - N_a} + 1}{\frac{N_b - N_a}{N_c - N_a} + 1 + \frac{2N_a}{N_c - N_a}}$$

$$= \frac{\frac{1}{h(N_i)}}{\frac{1}{h(N_i)} + \frac{2N_a}{(N_c - N_a)}} \qquad (11$$

Hence for a given hue a decrease in $N_a$ or increase in $(N_c$ will increase saturation.

The intensity $I$ of the choices made in fabricating the list of close-hued neighbors above also vary. As long as $N_c > N_a$, that is as long as we are not talking about gray, the value of $I$ will depend predominantly on $N_c$ or $N_b$. Thus in general

$$I(N_a, N_b, N_c + 1) > I(N_a, N_b, N_c) > I(N_a - 1, N_b - 1, N_c), \text{ and}$$

$$I(N_a, N_b + 1, N_c + 1) > I(N_a, N_b, N_c) > I(N_a - 1, N_b, N_c).$$

Thus the most intense and saturated colors of a given hue will have maximum $(N_c - N_a$

Just as has been done for hue, we can construct paths or lists comprising neighboring colors of nearly constant saturation. $N_c$ $N_a$

$$0 \leq \frac{1 - S(N_i)}{2} = \frac{N_a}{N_b + N_c} \leq \frac{1}{2}$$

Select a chosen saturation level, $S(N_i) = s_0$, and compute $\frac{1 - s_0}{2} = \frac{N_a}{N_b + N_c}$. We initially select a single value for $N_a$ $N_b$, and calculate a value for $N_c$ which will let $\frac{N_a}{N_a + N_c}$ be as close as possible to $\frac{1-S_o}{2}$. If this value for $N_c$ is greater than $n$, a lower value is selected for $N_a = N_b$. The highest $N_a$ value possible for a given saturation level is $N_a = n\left(\frac{1-S_o}{1+S_o}\right)$. Also notice that in the special case $S_o = 0$, any choice of $N_b$ and $N_c$ may be made. Now neighbors are found with the same value for $(N_a + N_c)$:

$$
\begin{gathered}
(N_a,\ N_a\ ,\ N_c\ ) \\
(N_a,\ N_a+1,\ N_c-1) \\
(N_a,\ N_a+2,\ N_c-2) \\
\vdots \\
(N_a,\ N_a+\ell,\ N_c-\ell)
\end{gathered}
$$

where $\ell = [\![\frac{1}{2}(N_c - N_b)]\!]$, so that $(N_c - \ell)$ is the smallest value which keeps the $N_c$ larger than the $N_b$. By reassigning the color values R, G, and B to a, b, and c above, six different lists of equal saturation colors with a given $N_a$ value may be constructed. Notice that higher values for $N_a$ result in higher values for $N_c$ and consequently in higher intensity levels overall. If we wish to find a list of colors with equal saturation and intensity, we proceed as follows:

Select $s_0$ and intensity $i_0$. Our first list entry will be $(N_a, N_a, N_c)$, whose intensity and saturation are:

$$I(N_a,N_a,N_c) = N_a + \frac{1}{\Delta}\log\left(10^{\Delta(N_c-N_a)} + 2\right) \qquad S(N_a,N_a,N_c) = \frac{N_c - N_a}{N_c + N_a}$$

To keep $\frac{N_a}{N_b+N_c}$ constant, for saturation, larger changes are necessary in $N_b + N_c$ than in $N_a$. On the other hand, keeping $I(N_i)$ constant will require larger changes in $N_a$ and $N_b$ than in $N_c$. We start, then, by increasing $N_b$:

$$I(N_a, N_a+1, N_c) = N_a + \frac{1}{\Delta}\log\left(10^{\Delta(N_c-N_a)} + 10^{\Delta} + 1\right) \qquad S(N_a,N_a+1,N_c) = \frac{N_c - N_a + 1}{N_c + N_a + 1}$$

Lowering $N_c$ will overcompensate for the intensity increment, and increasing $N_a$ will overcompensate for the saturation change. Clearly, changes in $N_c$ must be guided by intensity considerations, and changes in $N_a$ by saturation considerations.

$$\begin{array}{llll}
( N_a & , \; N_a & , \; N_c & ) \\
( N_a & , \; N_a+1 & , \; N_c & ) \\
( N_a & , \; N_a+2 & , \; N_c & ) \\
\;\;\vdots & \;\;\;\vdots & \;\;\vdots & \\
( \;\vdots & , \; N_a+(x-1) & , \; N_c & ) \\
( \;\vdots & , \; N_a+x & , \; N_c-1 & ) \\
 & \;\;\;\vdots & \;\;\vdots & \\
( N_a & & & ) \\
( N_a+1 & , \; N_a+y & , \; N_c-z & )
\end{array}$$

where $10^{\Delta x} = 10^{\Delta(N_c-N_a)}\left(1 - \frac{1}{10^{\Delta}}\right) + 1$ determines $x$, and $y = \frac{N_c}{N_a} + 1 + z$, where $z$ acts as an adjustment of $y$.

As the $N_c$ grow smaller and the $N_a$ larger, the value of $x$ grows smaller, and consequently $z$ may be greater. The values of $(y-z)$ will be smaller, as $N_c/N_a$ decreases; since $(y-z)$ will fall but $z$ will rise, $y$ may vary irregularly.

In the above formulation, the initial values of $s_0$ and $i_0$ are departed from in such a way that they may not be the exact center values of the resulting list of neighbors. Also, negligable errors may appear due to the neglect of changes in $N_a$, which are not included in the intensity calculation for $x$.

Around black and white, special conditions appear. Exposure levels can not be decreased or increased further, and hue and saturation are restricted.

$$\text{Black} = (0, 0, 0) \qquad \text{White} = (n, n, n)$$

The only neighbors of Black and White are the cases

$$N_c = 1,\ N_b = \begin{cases} 0 \text{ or} \\ 1 \end{cases},\ N_a = \begin{cases} 0 \text{ or} \\ 1 \end{cases};\quad N_a = n-1,\ N_b = \begin{cases} n \\ n-1 \end{cases},\ N_c = \begin{cases} n \\ n-1 \end{cases}$$

The cases $N_c = 1, N_b = 1, N_a = 1$ and $N_a = n-1, N_b = n-1, N_c = n-1$ are gray. The other two cases for each neighborhood compri[se] one primary and one secondary hue each.

<u>Definition</u>. A <u>monotonic list</u> is a list of neighboring colors, in which intensity $I(N_i)$ is never decreasing. That is, the $N_i$ always increase or stay constant.

We now construct a monotonic list which begins at black, ends at white, and which is also a list of neighbors of a chosen hue value, h. As we saw on page 8, a member of a list of neighbors of a chosen hue can have one of four successors:

$$\left.\begin{array}{lll}(N_a-1, & N_b-1, & N_c) \\ (N_a, & N_b, & N_c+1)\end{array}\right\} \text{case 1, for } \frac{(N_b-N_a)}{(N_c-N_a)\pm 1}$$

$$\left.\begin{array}{lll}(N_a-1, & N_b, & N_c) \\ (N_a, & N_b+1, & N_c+1)\end{array}\right\} \text{case 2, for } \frac{(N_b-N_a)\pm 1}{(N_c-N_a)\pm 1}$$

Each of these results in increasing saturation, and in an increase in the difference $(N_c-N_a)$. Of these, two only will increase intensity:

$$(N_a, N_b, N_c+1) \quad \text{or} \quad (N_a, N_b+1, N_c+1).$$

Selecting one of these two, using the procedure on page 8, a monotonic list is constructed which will terminate at $(N_a, N_b, n)$, when $N_c = n$. Notice that use of $(0,0,0)$ as a starting point means that the value of $N_a$ remains constant at zero. Following the list item $(0, N_b, n)$, the list can be extended by decreasing $(N_c-N_a)$, by decreasing saturation, by backtracking along the path of hue choices above, but making the choices which were rejected above — as these now take the forms $(N_a+1, N_b+1, N_c)$ and $(N_a+1, N_b, N_c)$.

The list item $(0, N_b, n)$ identifies the number of times case 2 appeared, namely $N_b$. The same number of instances will appear in the second portion of the list, which will consequently terminate at

$$(N_a', N_b', N_c')$$

where $N_a' = n$, $N_c' = n$, and $N_b' = N_b + (n - N_b) = n$. This list is monotonic, and ends at black, passing through the most saturated color of the particular hue. This monotonic list therefore simulates the shoulder and toe saturation which is characteristic of reversal color film.

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